FA18-BSE-048 Aeman Fatima

Assigment 3

$a = 0$ $b = 4$ $c = 8$

Putting values in equation

$$\int_1^3 ((8+1)x^2 \cos x + x^{4+1} - (0)x)\,dx$$

$$\int_1^3 (9x^2 \cos x + x^5)\,dx$$

where $n = 4$

1) Trapezoidal Rule

$$h = \frac{b-a}{n} = \frac{3-1}{4} = 0.5$$

Table

x	y
1	5.8627
1.5	9.0261
2	17.0187
2.5	52.5919
3	162.8106

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[y_1 + 2(y_2+y_3+y_4) + y_5\right] \quad \text{—(A)}$$

Putting values in (A)

$$\frac{0.5}{2}\left[5.8627 + 2(9.0261 + 17.0187 + 52.5919) + 162.8106\right]$$

$$\boxed{= 81.4867}$$

2) Simpson's 1/3rd Rule

$$h = \frac{b-a}{2n} = \frac{3-1}{2(4)} = 0.25$$

Table

x	y	
1	5.8627	y_1
1.25	7.4860	y_2
1.5	9.0262	y_3
1.75	11.5002	y_4
2	17.0187	y_5
2.25	29.0439	y_6
2.5	52.5919	y_7
2.75	94.3660	y_8
3	162.8106	y_9

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[y_1 + 4(y_2+y_4+y_6+y_8) + 2(y_3+y_5+y_7) + y_9\right] \quad \text{— (A)}$$

putting values in (A)

$$= \frac{0.25}{3}\left[5.8627 + 4(7.4860 \cancel{+ 9.0262} + 11.5002 + 29.0439 + 94.3660) + 2(9.0262 + 17.0187 + 52.5919) + 162.8106\right]$$

$$\boxed{= 74.6276}$$

Simpson's 3/8th Rule

$$h = \frac{b-a}{3n} = \frac{3-1}{3(4)} = 0.1666$$

Table

x	y	
1	5.8627	y_1
1.1666	6.9779	y_2
1.3332	7.9770	y_3
1.4998	9.0248	y_4
1.6664	10.4641	y_5
1.833	12.8542	y_6
1.9996	17.0058	y_7
2.1662	24.0118	y_8
2.3328	35.2728	y_9
2.4994	52.5166	y_{10}
2.666	77.8103	y_{11}
2.8326	113.5660	y_{12}
2.9992	162.5387	y_{13}

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[y_1 + 3(y_2+y_3+y_5+y_6+y_8+y_9+y_{11}+y_{12}) + 2(y_4+y_7+y_{10}) + y_{13})\right]$$

$$= \frac{3(0.1666)}{8}\left[5.8627 + 3(6.9779+7.977+10.4641+12.8542 + 24.0118+35.2728+77.8103+113.5662) + 2(9.0248 + 17.0058+52.5166)+162.5387\right]$$

$$\boxed{= 74.4900}$$

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Q NO 2)

Solving Analytically

$$\int_1^3 9x^2\cos x + x^5$$

$$= \left[\frac{x^6}{6} + 2(9)x\cos(x) + (9)(-2+x^2)\sin(x) \right]_1^3$$

$$= \left[\frac{x^6}{6}\right]_1^3 + 9\left[2\,[x\cos(x)]_1^3 + \left[(-2+x^2)\sin(x)\right]_1^3\right]$$

$$= \frac{1}{6}(3^6 - 1^6) + 9\left[2(3\cos(3) - \cos(1)) + (-2+3^2)\sin 3 - (-2+1^2)\sin(1)\right]$$

$$= \frac{1}{6}(728) + 9\left[2(-3.51028) + (1.8293)\right]$$

$$\boxed{= 74.6120}$$

Comparing results from Q NO 1

Analytical Solution	Trapezoidal	I_{S13}	I_{S38}
74.6120	81.4867	74.6276	74.4900
	A.E = 6.8747	A.E = 0.0156	A.E = 0.122

So Integration by Simpson's 1/3rd Rule is most efficient for this question.