Q) Find numerical solution of given ODE with Euler's Method, and compare solution with analytical Solution.

Given

$$\frac{dy}{dx} = \sin x$$

$h = 0.125$, $1.0472 \leq x \leq 2.0472$

$y(1.0472) = 2$

Solution

Formula

$$y_{n+1} = y_n + h[\sin(x_n)] \quad — (A)$$

$$x_{n+1} = x_n + h \quad — (B)$$

For y_1

$$y_1 = y_0 + h[\sin(x_0)]$$

$$y_1 = 2 + (0.125)[\sin(1.0472)]$$

$$\boxed{y_1 = 2.1083}$$

For x_1

$$x_1 = x_0 + h$$

$$x_1 = 1.0472 + 0.125$$

$$\boxed{x_1 = 1.1722}$$

Table

	x	y	
x_0	1.0472	2	y_0
x_1	1.1722	2.1083	y_1
x_2	1.2972	2.2235	y_2
x_3	1.4222	2.3438	y_3
x_4	1.5472	2.4688	y_4
x_5	1.6722	2.5931	y_5
x_6	1.7972	2.7149	y_6
x_7	1.9222	2.8323	y_7
x_8	2.0472	2.9434	y_8

Comparing with Analytical Answers

Formula

$$y(x) = -\cos x + 2.5$$

Comparison table:

n	x	x+y'	y(x)	Error
0	1.0472	2	2	0.000
1	1.1972	2.1083	2.1119	0.0360
2	1.8972	2.2235	2.2298	0.0063
3	1.4222	2.3438	2.3519	0.0081
4	1.5472	2.4688	2.4764	0.0076
5	1.6722	2.5931	2.6012	0.0081
6	1.7972	2.7149	2.7245	0.0096
7	1.9222	2.8323	2.8442	0.0119
8	2.0472	2.9434	2.9586	0.0152