INTERPOLATION:

EQUAL SPACE DATA:

x	
0	
0.5	$h=0.5$
1	
1.5	
2	

t	
1	
2	$h=1$
3	
4	
5	

x	
1	
3	$h=2$
5	
7	

If the h is same for all terms then tha data is equaly spaces.

$$h = x_{n+1} - x_n$$

There are 3 methods.

1, Newton forword
2, Newton Backword
3, Stirling

Newton Forword Interpolation formula:-

$\Rightarrow$ x is given $x = \_\_$ point of interpolation.

$$f(x) - P(p) = \underbrace{y_0}_{} + p\underbrace{\Delta y_0}_{\text{1st difference of } y_0} + \frac{p(p-1)}{2!}\underbrace{\Delta^2 y_0}_{\text{2nd difference of } y_0} + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \dots$$

where

$$p = \frac{x - x_0}{h}$$

Difference Table:-

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	y_0			
		$y_1 - y_0 = \Delta y_0$		
x_1	y_1		$\Delta y_1 - \Delta y_0 = \Delta^2 y_0$	
		$y_2 - y_1 = \Delta y_1$		$\Delta^2 y_1 - \Delta^2 y_0 = \Delta^3 y$
x_2	y_2		$\Delta y_2 - \Delta y_1 = \Delta^2 y_1$	
		$y_3 - y_2 = \Delta y_2$		
x_3	y_3			

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	y_0			
		Δy_0		
x_1	y_1		$\Delta^2 y_0$	
		Δy_1		$\Delta^3 y_0$
x_2	y_2		$\Delta^2 y_1$	
		Δy_2		
x_3	y_3			

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_0 1	3 y_0					
$\boxed{x=1.5}$		3 Δy_0				
x_0 2	6 y_0		8 $\Delta^2 y_0$			
$x=2.25$		11 Δy_0		6 $\Delta^3 y_0$		
3	17		14 $\Delta^2 y_0$		0	
		25		6 $\Delta^3 y_0$		0
4	42		20		0	
		45		6		
5	87		26			
		71				
6	158					

⇒ If the data become constant at any column so it is generated by a polinomial

⇒ first you have to see where the point lies and the its upper entry is consider as x_0.

⇒ And the values $y_0, \Delta y_0, \Delta^2 y_0$ are digonaly downword from x_0.

Q1: Interpolat $x = 1.5$ for above data.

$$P(x) = f(p) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 \quad \text{(A)}$$

$$p = \frac{x - x_0}{h} = \frac{1.5 - 1}{1} = 0.5$$

put values in above formula.

$$P(1.5) = f(0.5) = 3 + 0.5(3) + \frac{0.5(0.5-1)(8)}{2!} + \frac{0.5(0.5-1)(0.5-2)(6)}{3!}$$

$$P(1.5) = 3 + 1.5 + (-0.125)(8) + (0.06250)(6)$$
$$= 3 + 1.5 + (-1) + 0.37500$$
$$= 3.875$$

Newton Backword Interpolation formula:-

independent, independant, Backword sign (1st Backword difference of y_0)

$$P(x) = P(p) = f(x) = y_0 + p\,\nabla y_0 + \frac{p(p+1)}{2!}\nabla^2 y_0 + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_0$$

$+ \ldots + (n-1)$th difference colum. —— (B)

where $p = \dfrac{x - x_0}{h}$ —— (A)

Q No1. Interpolat $x = 5.5$ / Comput y at $x = 5.5$ for given data set

x	y	Δy_0	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1	3					
		3				
2	6		8			
		11		6		
3	17		14		0	
		25		6		0 $\nabla^5 y_0$
4	42		20		0 $\nabla^4 y_0$	
		45		6 $\nabla^3 y_0$		
5	87		26 $\nabla^2 y_0$			
$x = 5.5$		71				
6 x_0	158 y_0	∇y_0				

=> If the point chosen such as 5.5 if we use forword interpolation we will get only 2 points which donot give more accurate answer.

=> In backword formula chose 6 ~~as~~ lower as x_0 and move backword.

Sol:-

$$P(x) = y_0 + P\nabla y_0 + \frac{P(P+1)}{2!}\nabla^2 y_0 + \frac{P(P+1)(P+2)}{3!}\nabla^3 y_0 + \frac{P(P+1)(P+2)(P+3)}{4!}\nabla^4 y_0$$

$$P = \frac{5.5-6}{1} = -0.5$$

$$\cancel{P(55) = 158 + \frac{(-0.5)(-0.5+1)}{2!}}$$

$$P(55) = 158 + (-0.5)(71) + \frac{(-0.5)(-0.5+1)(\cancel{26}^{13})}{\cancel{2!}\,1} + \frac{(-0.5)(-0.5+1)(-0.5+2)(\cancel{6})}{\cancel{6}} + 0$$

$$P(55) = 118.875$$

Q2: Interpolate $x = 5.75$ for given data set

Stirling Interpolation formula

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1	3					
		3				
2	6		8			
		11	$\delta^2 y_0$	6		
x_0 3	y_0 17	$\mu \delta y_0$	14	$\mu \delta^3 y$	0 $\delta^4 y_0$	
$x = 3.25$		25		6		0
4	42		20		0	
		45		6		
5	87		26			
		71				
6	158					

=> In central formula we get the point in straight line and the number of odd values will be calculatey by taking mean of upper and lower values.

$$P(x) = y_0 + p\mu\delta y_0 + \frac{p^2}{2!}\delta^2 y_0 + \frac{p(p^2-1)}{3!}\mu\delta^3 y_0 + \frac{p^2(p^2-1)}{4!}\delta^4 y_0 +$$

$$\frac{p(p^2-1)(p^2-2)}{5}\mu\delta y_0 + \frac{p^2(p^2-1)(p^2-2)}{6!}\delta y_0$$

21.703

Q No: Interpolate $x = 3.25$ for above data

Sol:-

$$\mu\,\delta y_0 = \frac{11 + 25}{2} = 18$$

$$\mu\,\delta^3 y_0 = \frac{6 + 6}{2} = 6$$

$$P = \frac{x - x_0}{h} = \frac{3.25 - 3}{1} = 0.25$$

$$P(3.25) = y_0 + P\mu\,\delta y_0 + p^2\frac{\delta^2 y_0}{2!} + \frac{p(p^2-1)}{3!}\mu\,\delta^3 y_0 + \frac{p^2(p^2-1)}{4!}\delta^4 y_0$$

$$= 17 + (0.25)(18) + \frac{(0.25)^2(\cancel{14})^{7}}{\cancel{2!}} + \frac{(0.25)((0.25)^2-1)(\cancel{6})}{\cancel{6}} + \frac{(0.25)^2((0.25)^2-1)(0)}{4!}$$

$$= 21.703$$

Q: Interpolate (a) $x = 1.25$ (Δ) (b) $x = 2.75$ (∇) (c) $x = 2.12$ (δ)

x	y
1	4
1.5	8.8125
2	13
2.5	30.8125
3	66