1 Dolittle's Method:-

is used to decompose matrix A into upper triangular and lower triangular where the digonal of Lower triangular is ristricted to 1.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}, \quad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$$ Decompose matrix A into L, U decomposition using Dolittle Method.

Sol

$$A = L \times U$$

$$\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix} \times \begin{bmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} u_{11} + 0 & u_{12} + 0 \\ l_{21}u_{11} + 0 & l_{21}u_{12} + u_{22} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} \end{bmatrix}$$

$u_{11} = 2$, $u_{12} = 4$

$l_{21}u_{11} = 3$ $\quad$ $l_{21}u_{12} + u_{22} = 1$

$l_{21}(2) = 3$ $\quad$ $\left(\frac{3}{2}\right)(4) + u_{22} = 1$

$l_{21} = \frac{3}{2}$ $\quad$ $\left(\frac{12}{2}\right) + u_{22} = 1$

$u_{22} = 1 - \left(\frac{12}{2}\right) = 1 - 6 = -5.$

$$L = \begin{bmatrix} 1 & 0 \\ 3/2 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 2 & 0 \\ 4 & -5 \end{bmatrix}$$

Decomposition of matrix A.

Verification

$LU = A$

$$\begin{bmatrix} 1 & 0 \\ 3/2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 2+0 & 4+0 \\ (3/\cancel{2})\cancel{2}+0 & \frac{3}{\cancel{2}}(\cancel{4})^{2}+1(-5) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$$

Decompose

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 5 & 3 \\ 4 & 3 & 6 \end{bmatrix}.$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}, \qquad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

CROUT'S METHOD:-

Crouts method is used to decompose a square metrix into upper and lower matrix. where the digonal matrix of upper matrix is equal to 1.

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \qquad U = \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

Q:

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 4 & 2 \\ 2 & 3 & 5 \end{bmatrix}$$

Sol:-

$$A = L \times U$$

by using crouts method.

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 4 & 2 \\ 2 & 3 & 5 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{12} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix}$$

$\boxed{l_{11} = 2}$ $\boxed{l_{21} = 3}$ $\boxed{l_{31} = 2}$

$\cancel{l_{11} U_{12} = 1}$ $l_{11} U_{12} = 1$ $l_{11} U_{13} = 1$

$\cancel{l_{11}(3) = 1}$ $(2) U_{12} = 1$ $(2) U_{13} = 1$

$\cancel{\boxed{l_{11} = 1/3}}$ $\boxed{U_{12} = 1/2}$ $\boxed{U_{13} = 1/2}$

$l_{21} U_{12} + l_{22} = 4$

$(3)(1/2) + l_{22} = 4$

$(3/2) + l_{22} = 4$

$l_{22} = 4 - 3/2$

$\boxed{l_{22} = 5/2}$

$l_{21} U_{12} + l_{22} U_{23} = 2$

$(3)(1/2) + 5/2 \, U_{23} = 2$

$3/2 + 5/2 \, U_{23} = 2$

$5/2 \, U_{23} = 2 - 3/2$

$5/2 \, U_{23} = 1/2$

$U_{23} = 1/2 \times 2/5$

$\boxed{U_{23} = 1/5}$

$l_{31} U_{12} + l_{32} = 3$

$(2)(1/2) + l_{32} = 3$

$1 + l_{32} = 3$

$l_{32} = 3 - 1$

$\boxed{l_{32} = 2}$

$l_{31} U_{13} + l_{32} U_{23} + l_{33} = 5$

$(2)(1/2) + (2)(1/5) + l_{33} = 5$

$1 + 2/5 + l_{33} = 5$

$7/5 + l_{33} = 5$

$l_{33} = 5 - 7/5$

$\boxed{l_{33} = \frac{18}{5}}$

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}, \quad U = \begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$L = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 5/2 & 0 \\ 2 & 2 & 18/5 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 1/2 & 1/2 \\ 0 & 1 & 1/5 \\ 0 & 0 & 1 \end{bmatrix}$$

"Solve System of Liner Equation by L U decomposition."

$$Ax = B \text{ —— } ① \ \because$$

$\because A = LU$

Put in ①

$$LUx = B \text{ —— } ②$$

Let $Ux = y$ —— ④ $\qquad x_{m\times 1} \qquad y_{m\times 1}$

$$Ly = B \text{ —— } ③$$

Solve the following System of Liner Equation by using do little technique.

$$2x_1 - 2x_2 + 4x_3 = 5$$
$$-2x_1 + x_2 + 2x_3 = 10$$
$$2x_1 + x_2 + 2x_3 = 15$$

Sol:- As we know that System of liner eq. can be writter as.

$$Ax = b$$

where

$$A = \begin{bmatrix} 2 & -2 & 4 \\ -2 & 1 & 2 \\ 2 & 1 & 2 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 10 \\ 15 \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

y have the same order as x...

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -3 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 2 & -2 & 4 \\ 0 & -1 & 6 \\ 0 & 0 & 16 \end{bmatrix}$$

Now $Ly = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 15 \end{bmatrix}$$

$y_1 = 5 \longrightarrow (1)$

$-y_1 + y_2 = 10 \longrightarrow (2)$

$y_1 - 3y_2 + y_3 = 15 \longrightarrow (3)$

$y_1 = 5$

by forword substitution

$(2) \longrightarrow -1(5) + y_2 = 10$

$-5 + y_2 = 10$

$y_2 = 10 + 5$

$y_2 = 15$

$(3) \longrightarrow (5) - 3(15) + y_3 = 15$

$5 - 45 + y_3 = 15$

$-40 + y_3 = 15$

$y_3 = 15 + 40$

$y_3 = 55$

$$y = \begin{bmatrix} 5 \\ 15 \\ 55 \end{bmatrix}$$

$Ux = y$

$$\begin{bmatrix} 2 & -2 & 4 \\ 0 & -1 & 6 \\ 0 & 0 & 16 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \\ 55 \end{bmatrix}$$

$$2x_1 - 2x_2 + 4x_3 = 5$$
$$-x_2 + 6x_3 = 15$$
$$16x_3 = 55$$

by applying backword substitution.

$$x_3 = 55/16$$

② → $-x_2 + 6(55/16) = 15$

$\cancel{-x_3 + 330 = 15}$

$$-x_2 + 6(55/16) = 15$$
$$-x_2 + 330/16 = 15$$
$$-x_2 = 15 - 330/16$$
$$-x_2 = -45/8$$
$$x_2 = 45/8$$

③ → $\cancel{2(55/16)}$

③ → $2x_1 - \cancel{2}^{1}(45/\cancel{8}_{4}) + \cancel{4}^{1}(55/\cancel{16}_{4}) = 5$

$$2x_1 (-45/4) + (55/4) = 5$$
$$2x_1 + 5/2 = 5$$
$$2x_1 = 5 - 5/2$$
$$x_1 = 5/2\times2$$
$$x_1 = 5/4$$

the Solution is $(x_1, x_2, x_3) = \left(\frac{5}{4}, \frac{45}{8}, \frac{55}{16} \right)$

$$x = \begin{bmatrix} 5/4 \\ 45/8 \\ 55/16 \end{bmatrix}$$

Q Solve the following System of liner equation by using crouts LU decomposition method.

$$L = \begin{bmatrix} 2 & 0 & 0 \\ -2 & -1 & 0 \\ 2 & 3 & 16 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -2 & 4 \\ -2 & 1 & 2 \\ 2 & 1 & 2 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 10 \\ 15 \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Applying crouts LU Decomposition.

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \qquad U = \begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = L \times U$$

$$\begin{bmatrix} 2 & -2 & 4 \\ -2 & 1 & 2 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -2 & 4 \\ -2 & 1 & 2 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}U_{12} & l_{11}U_{13} \\ l_{21} & l_{21}U_{12} + l_{22} & l_{21}U_{13} + l_{22}U_{23} \\ l_{31} & l_{31}U_{12} + l_{32} & l_{31}U_{13} + l_{32}U_{23} + l_{33} \end{bmatrix}$$

$$l_{11} = 2 \qquad l_{21} = -2 \qquad l_{31} = 2$$

$$l_{11}U_{12} = -2 \qquad\qquad l_{11}U_{13} = 4$$

$$(2)U_{12} = -2 \qquad\qquad (2)U_{13} = 4$$

$$U_{12} = -2/2 \qquad\qquad U_{13} = 4/2$$

$$\boxed{U_{12} = -1} \qquad\qquad \boxed{U_{13} = 2}$$

$$l_{21}u_{12} + l_{22} = 1$$
$$(-2)(-1) + l_{22} = 1$$
$$2 + l_{22} = 1$$
$$l_{22} = 1 - 2$$
$$\boxed{l_{22} = -1}$$

$$l_{31}u_{12} + l_{32} = 1$$
$$(2)(-1) + l_{32} = 1$$
$$-2 + l_{32} = 1$$
$$l_{32} = 1 + 2$$
$$\boxed{l_{32} = 3}$$

$$l_{31}u_{13} + l_{32}u_{23} + l_{33} = 2$$
$$(2)(2) + (3)(-6) + l_{33} = 2$$
$$4 - 18 + l_{33} = 2$$
$$-14 + l_{33} = 2$$
$$l_{33} = 2 + 14$$
$$\boxed{l_{33} = 16}$$

$$l_{21}u_{13} + l_{22}u_{23} = 2$$
$$(-2)(2) + (-1)u_{23} = 2$$
$$-4 - u_{23} = 2$$
$$-u_{23} = 2 + 4$$
$$\boxed{u_{23} = -6}$$

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ -2 & -1 & 0 \\ 2 & 3 & 16 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

Now $Ly = b$

$$\begin{bmatrix} 2 & 0 & 0 \\ -2 & -1 & 0 \\ 2 & 3 & 16 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 15 \end{bmatrix}$$

$$2y_1 = 5 \longrightarrow ①$$
$$-2y_1 - y_2 = 10 \longrightarrow ②$$
$$2y_1 + 3y_2 + 16y_3 = 15 \longrightarrow ③$$

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By using forword Substitution.

① $\longrightarrow$ $2y_1 = 5$

$$\boxed{y_1 = 5/2}$$

② $\longrightarrow$ $-2(5/2) - y_2 = 10$

$$-5 - y_2 = 10$$

$$-y_2 = 10 + 5$$

$$\boxed{y_2 = -15}$$

③ $\longrightarrow$ $2(5/2) + 3(-15) + 16y_3 = 15$

$$5 - 45 + 16y_3 = 15$$

$$-40 + 16y_3 = 15$$

$$16y_3 = 15 + 40$$

$$\boxed{y_3 = 55/16}$$

$$y = \begin{bmatrix} 5/2 \\ -15 \\ 55/16 \end{bmatrix}$$

Now $Ux = y$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5/2 \\ -15 \\ 55/16 \end{bmatrix}$$

$x_1 - x_2 + 2x_3 = 5/2$ ——— ④

$x_2 - 6x_3 = -15$ ——— ⑤

$x_3 = 55/16$ ——— ⑥

By using backword substitution.

⑥ $\longrightarrow$ $\boxed{x_3 = \frac{55}{16}}$

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