# JACOBI ITERATIVE METHOD

Let we have 3x3 system of Linear eq.

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$ ——— (i)

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$ ——— (ii)

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$ ——— (iii)

**Step 1:-**

If the diagonal elements of above equations are not diagonal dominant element then try to swap the rows and make them diagonal dominant.

**Step 2:-**

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{11}x_1 = b_1 - a_{12}x_2 - a_{13}x_3$$

$$x_1 = \frac{b_1 - a_{12}x_2 - a_{13}x_3}{a_{11}}$$

Similarly:

$$x_2 = \frac{b_2 - a_{21}x_1 - a_{23}x_3}{a_{22}}$$

$$x_3 = \frac{b_3 - a_{31}x_1 - a_{32}x_2}{a_{33}}$$

from above equations we will get values of $x_1, x_2, x_3$.

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Step 3:-

Apply ~~iteration~~, iterative process/function method on equation of $x$.

$$x_1^{(n+1)} = \frac{\left(b_1 - a_{12}x_2^{(n)} - a_{13}x_3^{(n)}\right)}{a_{11}}$$

$$x_2^{(n+1)} = \frac{\left(b_2 - a_{21}x_1^{(n)} - a_{23}x_3^{(n)}\right)}{a_{22}}$$

$$x_3^{(n+1)} = \frac{\left(b_3 - a_{31}x_1^{(n)} - a_{32}x_2^{(n)}\right)}{a_{33}}$$

Step 4:-

If the initial point/values is not given then we always uses $x = (0, 0, 0)$ or.

$x_1 = 0$ , $x_2 = 0$ , $x_3 = 0$

Q Solve the system of Linear equation by using Jacobi iterative method.

$$4x_1 - x_2 + 2x_3 = 10$$

$$x_1 + 5x_2 - 3x_3 = 12$$

$$2x_1 + 2x_2 - 6x_3 = 14$$

Step 1:-

As we see that the given system is diagonal dominant.

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**Step 2:-**

Making equations.

$$x_1 = \frac{10 + x_2 - 2x_3}{4}$$

$$x_2 = \frac{12 - x_1 + 3x_3}{5}$$

$$x_3 = \frac{-14 + 2x_1 + 2x_2}{6}$$

**Step-3:-**

Making iterative function

$$x_1^{(n+1)} = \frac{10 + x_2^{(n)} - 2x_3^{(n)}}{4}$$

$$x_2^{(n+1)} = \frac{12 - x_1^{(n)} + 3x_3^{(n)}}{5}$$

$$x_3^{(n+1)} = \frac{-14 + 2x_1^{(n)} + 2x_2^{(n)}}{6}$$

**Step 4:-**

Starting point / Value:

As not given in question So,

$$x_1^{(0)} = 0 \ , \ x_2^{(0)} = 0 \ , \ x_3^{(0)} = 0$$

Now we put $n = 0$.

$$x_1^{(1)} = \frac{10 + x_2^{(0)} - 2x_3^{(0)}}{4}$$

$$x_1^{(1)} = \frac{10 + 0 + 0}{4} = 2.5000$$

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$$x_2^{(1)} = \frac{12 - x_1^{(0)} + 3x_3^{(0)}}{5}$$

$$x_2 = \frac{12 - 0 + 0}{5}$$

$$x_2 = 2.4000$$

$$x_3^{(1)} = \frac{-14 + 12x_1^{(0)} + 12x_2^{(0)}}{6}$$

$$x_3 = \frac{-14 + 0 + 0}{6}$$

$$x_3 = -2.3333$$

| n | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| - | 0 | 0 | 0 |
| 0 | 2.5000 | 2.4000 | -2.3333 |
| 1 | 4.26665 | 0.50002 | -0.7000 |
| 2 | 2.97500 | 1.12667 | -0.7444 |
| 3 | 3.15387 | 1.35836 | -0.96611 |
| 4 | 3.32264 | 1.18956 | -0.82926 |
| 5 | 3.21202 | 1.23792 | -0.82927 |
| 6 | 3.22411 | 1.26003 | -0.85002 |

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# Gauss Seidel Itrative Method

This method is an advanced form of

Conditions:-

1) It must be digonal dominent.
2) $eq_1 \rightarrow x_1$
   $eq_2 \rightarrow x_2$
   $eq_3 \rightarrow x_3$
3) Initial guess is 0 if values are not given.
4) Initial function:-

$$eq_1 \rightarrow x_1^{(n+1)} = \frac{(b_1 - a_{12}x_2^{(n)} - a_{13}x_3^{(n)})}{a_{11}}$$

$$eq_2 \rightarrow x_2^{(n+1)} = \frac{(b_2 - a_{21}x_1^{(n+1)} - a_{23}x_3^{(n)})}{a_{22}}$$

$$eq_3 \rightarrow x_3^{(n+1)} = \frac{(b_3 - a_{31}x_1^{(n+1)} - a_{32}x_2^{(n+1)})}{a_{33}}$$

⇒ In jecobe method for calculating $x_2$, $x_3$ we use the same previous values but in gauss seidel method for $x_2$ we will us updated values of $x_1$ and similarly for $x_3$.

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$$4x_1 - x_2 + 2x_3 = 10$$
$$x_1 + 5x_2 - 3x_3 = 12$$
$$2x_1 + 2x_2 - 6x_3 = 14$$

Sol

Step 1:- The system is digonal dominent.

Step 2:- Let $x_1 = 0$, $x_2 = 0$ $x_3 = 0$ as not given in Question.

$$x_1^{(n+1)} = \frac{(10 + x_2^{(n)} - 2(x_3^{(n)}))}{4} = \frac{10 + 0 - 2(0)}{4} = \frac{10}{4} = \frac{5}{2} = 2.5$$

$$x_2^{(n+1)} = \frac{(12 - x_1^{(n+1)} + 3x_2^{(n)})}{5} = \frac{12 - (\frac{5}{2}) + 3(0)}{5} = 1.9$$

$$x_3^{(n+1)} = \frac{(-14 + 2x_1^{(n+1)} + 2x_2^{(n+1)})}{6} = \frac{-14 + 2(2.5) + 2(1.9)}{6} = -0.8667$$

| $n$ | $x_1^{(n)}$ | $x_2^{(n)}$ | $x_3^{(n)}$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 2.5 | 1.9 | -0.8667 |
| 2 | 3.408 | 1.19838 | -0.7978 |
| 3 | 3.1985 | 1.2816 | -0.84 |
| 4 | 3.2404 | 1.247 | -0.8372 |
| 5 | 3.2306 | 1.2511 | -0.8393 |

3.232 1.25 -0.8393

=> Stop when all columns get repeated values....